WAVEGUIDE PROPERTIES OF GRADIENT PERIODIC FIBER LIGHTGUIDES

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Propagation of lower surface waves is studied in gradient fibers with longitudinal additive inhomogeneity.

In recent years various optoelectronic applications have been developed for fiber and film lightguides having a gradient (inhomogeneous) change in their index of refraction, which may be periodic. Among these are use in resonant structures with distributed inverse feedback, input and output of collimated laser beams, etc. [1-7]. Theoretical analysis of gradient lightguides presents great mathematical difficulties because many singular features of the general theory of linear equations with variable coefficients ahve been studied only for special classes of equations such as Weber, Bessel, Mathieu, Meinser, Lamet, Gauss equations, etc. However, practical application of various gradient media demands study of new equations with periodic coefficients.

At the present time wave propagation has been studied in media with a gradient distribution of material characteristics in a single direction [2, 4, 6, 8-11]. It is of importance to accurately consider the waveguide properties of isotropic media and waveguides based on such media, the dielectric permittivity of which can simultaneously change in different directions with respect to the wave propagation direction.

We will consider passage of waves through dielectric fibers of circular cross section with inhomogeneous periodic properties along the longitudinal coordinate z and a gradient profile over the cross section with azimuthal symmetry (Fig. 1). We will apply Maxwell's equations for monochromatic fields directly, producing two independent systems for E and H waves. In a cylindrical coordinate system these equations have the following form:

$$\frac{\partial E_\varphi}{\partial z} = j\omega\mu_0 H_r, \quad \frac{1}{r}\frac{\partial}{\partial r}(rE_\varphi) = -j\omega\mu_0 H_z,$$
$$\frac{\partial H_r}{\partial z} - \frac{\partial H_z}{\partial r} = j\omega\varepsilon(r, z) E_\varphi, \tag{1}$$

$$-\frac{\partial H_\varphi}{\partial z} = j\omega\varepsilon(r, z) E_r, \quad \frac{1}{r}\frac{\partial}{\partial r}(rH_\varphi) = j\omega\varepsilon(r, z) E_z,$$
$$\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = -j\omega\mu_0 H_\varphi, \tag{2}$$

where μ_0, $\varepsilon(r, z)$ are the magnetic permittivity of a vacuum and the dielectric permittivity of the gradient core material.

We will consider the propagation of surface H waves. From system (1) we obtain, for the transverse component of the electric field

$$\frac{\partial^2 E_\varphi}{\partial r^2} + \frac{1}{r}\frac{\partial E_\varphi}{\partial r} + \frac{\partial^2 E_\varphi}{\partial z^2} + \left[\omega^2\mu_0\varepsilon(r, z) - \frac{1}{r^2}\right] E_\varphi = 0. \tag{3}$$

We will consider the two-dimensional spatial profile of the core dielectric permittivity in the form

$$\varepsilon(r, z) = \varepsilon(0)(1 - ar^2 + br^4 - 2q\cos\beta z), \tag{4}$$

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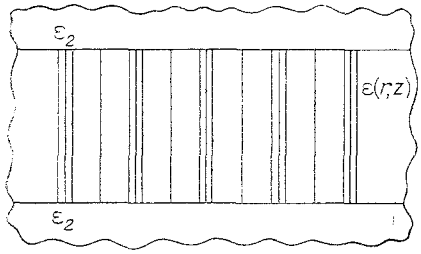

Fig. 1. Schematic diagram of gradient periodic fiber lightguide.

where $\varepsilon(0)$ is the dielectric permittivity value at the geometric axis of the core and q is the modulation coefficient, $0 \le q < 1$.

If the solutions of Eq. (3) are constructed, the other components are then found by simple differentiation of Eq. (1). We will seek a solution of Eq. (3) in the form R(r)Z(z):

$$\frac{d^2R}{dr^2} + \frac{1}{r}\frac{dR}{dr} + \left[\omega^2\mu_0\varepsilon(r) - \gamma^2 - \frac{1}{r^2}\right]R = 0, \tag{5}$$

$$\frac{d^2Z}{dz^2} + (\gamma^2 - 2qk^2\cos\beta z)\,Z = 0. \tag{6}$$

On the basis of the results of [12-14], we write the general solution of Eq. (5) in the following manner:

$$R(r) = A_1R_1(a,\ b,\ \varkappa r) + A_2R_2(a,\ b,\ \varkappa r), \tag{7}$$

where $R_1(a,\ b,\ \varkappa r)$, $R_2(a,\ b,\ \varkappa r)$ are special wave optics functions.

By introducing new variables in the form $\beta z = 2y$ Eq. (6) reduces to a Mathieu equation

$$\frac{d^2Z}{dy^2} + (\alpha - 2q_1\cos 2y)\,Z = 0, \tag{8}$$

where $\alpha = 4\gamma^2/\beta^2$, $q_1 = 4k^2q/\beta^2$, $\beta = 2\pi/T_b$. Using the results of [15, 16], we write the solution of Eq. (8):

$$Z(y,\ \varkappa) = B_1\Theta_1(\varkappa,\ y)\,e^{-j\eta y} + B_2\Theta_2(\varkappa,\ y)\,e^{j\eta y}, \tag{9}$$

where η is the characteristic index or wave number of directed surface waves along the coordinate y; $\Theta_1(\varkappa,\ y)$ and $\Theta_2(\varkappa,\ y)$ are Mathieu functions.

On the basis of Eqs. (7) and (9) we find expressions for the azimuthal electric field components on the form

$$E_\varphi = D_1R_1(a,\ b,\ \varkappa r)\,\Theta_1(\varkappa,\ y)\,e^{-j\eta y}. \tag{10}$$

Here we consider the finite field value on the axis of the gradient periodic lightguide. The other magnetic field components are found from Eq. (1):

$$\begin{aligned} H_r &= \frac{D_1}{j\omega\mu_0}R_1(a,\ b,\ \varkappa r)\,[\Theta'_{1,y}(\varkappa,\ y)\,e^{-j\eta y} - j\eta\Theta_1(\varkappa,\ y)\,e^{-j\eta y}],\\ H_y &= -\frac{\varkappa D_1}{j\omega\mu_0}\left[R'_{1,r}(a,\ b,\ \varkappa r) + \frac{1}{\varkappa}R_1(a,\ b,\ \varkappa r)\right]\Theta_1(\varkappa,\ y)\,e^{-j\eta y}. \end{aligned} \tag{11}$$

In an external homogeneous infinite cylindrical layer Maxwell's equations transform to a Bessel equation and a special form of the Mathieu equation with $q_1 = 0$ (harmonic oscillation equation), the solution of which is well known:

$$\check{E}_\varphi = C_1K_1(\sigma r)\,e^{-j\nu y}. \tag{12}$$

The remaining electromagnetic field components are easily found from Eq. (1).

The relationship between the wave numbers in the outer and inner layers has the following form: $\varkappa^2 = k^2 - \gamma^2$, $k^2 = \omega^2\mu_0\varepsilon(0)$, $-\sigma^2 = k^2(1-\varepsilon) - \varkappa^2$. Here $\varepsilon = \varepsilon_2/\varepsilon(0)$, while ε_2 is the relative dielectric permittivity of the infinite shell.

The propagation constant η of the directed waves is related to the eigennumber γ of Eq. (8) through expressions obtained by substituting Eq. (9) in the Mathieu equation [2]. After normalizing wave numbers, we obtain

$$\varkappa^2 + \sigma^2 = k^2(1 - \check{\varepsilon}) = \pi^2 \tilde{d}^2, \quad \sigma = (\pi^2 \tilde{d}^2 - \varkappa^2)^{1/2}, \quad k^2 = \frac{\pi^2 \tilde{d}^2}{1 - \check{\varepsilon}}.$$

These last expressions can easily be related to the coefficients of Eq. (8).

To study the dispersion properties of the gradient periodic waveguide we make use of the conditions of continuity of the longitudinal and azimuthal components of the electric and magnetic fields at the boundary between the internal and external media. Substituting in these conditions Eqs. (10)-(12), we obtain a transcendental equation for definition of the transverse and longitudinal wave numbers

$$\frac{\varkappa R'_{1,r}(a,\ b,\ \varkappa)}{R_1(a,\ b,\ \varkappa)} + \frac{\sigma K_0(\sigma)}{K_1(\sigma)} = 0. \tag{13}$$

To determine the wave numbers of surface waves directed by the gradient periodic fiber lightguide, we substitute the first expression of Eq. (9) in Eq. (8). Then, considering an expansion of the form

$$\Theta_1(\varkappa,\ y) = \sum_{n=-\infty}^{\infty} c_n(\varkappa) \exp(-j2ny), \tag{14}$$

we obtain

$$\sum_{n=-\infty}^{\infty} c_n(\varkappa)\,[B_n - 2\cos 2y]\, e^{j2ny} = 0. \tag{15}$$

Writing the periodic term in the last expression in exponential form, we obtain a recursion relationship [2, 16, 17-19] for determination of η_n with known values of the longitudinal wave numbers, mean period value, dielectric permittivity spatial profile amplitude, and wave numbers in a free space with material characteristics $\varepsilon(0)$, μ_0:

$$B_n c_n(\varkappa) + c_{n+1}(\varkappa) + c_{n-1}(\varkappa) = 0$$

or, in other words, a stability diagram (Fig. 2).

In the stability regions the wave number is real, and on the boundaries of these regions it takes on the values $\eta = l,\ l = 0, 1, 2, \ldots,\ l \leqslant \eta < l + 1,\ \operatorname{Im}\eta = 0,\ \operatorname{Re}\eta \geqslant 0$.

The solution of Eq. (8) is limited here. In regions of instability $\pm\eta = \eta' + j\eta'' = l + j\check{a}$, where $\check{a}$ is a positive number. The solution of Eq. (8) increases without limit at infinity. In [15] it was shown that in this case the expression $\Theta(\varkappa,\ y)\exp(j\eta_n y)$ is either an entirely real of an entirely imaginary function, i.e., depending on location in the first or second type of region in the stability diagram, wave propagation occurs or the waveguide system loses these properties. The gradient parameters of the transverse spatial profile of the dielectric permittivity qualitatively and quantitatively determine the transition from one region of the stability diagram to the other. It can easily be noted from the stability diagram that the geometric locus of the longitudinal wave numbers will be almost a straight line, the form and position of which depend on the dispersion curve of the transverse wave number (Fig. 2). For values of dimensionless diameter outside the limits of the surface wave critical region, the dispersion characteristic forms an angle with the axis equal to

$$\Psi = \arctan \frac{q}{(1 - \varkappa^2 k^{-2})}. \tag{16}$$

When one or the other surface wave is considered, the dispersion characteristic slides along the stability diagram, remaining practically parallel to its former position, permitting us to find concrete values for the longitudinal wave numbers of symmetric surface waves. The initial point corresponding to the critical dimensionless diameter lies upon a straight

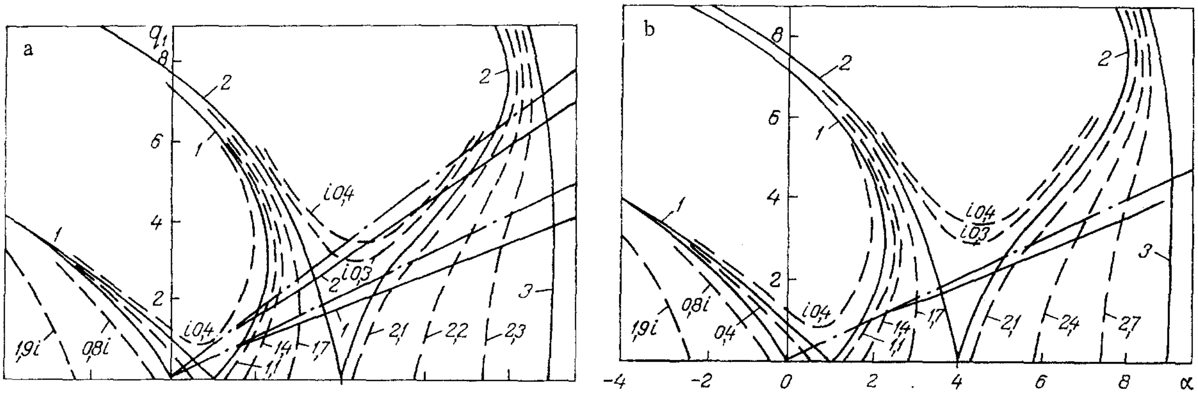

Fig. 2. Stability diagram for surface H_{01} wave of gradient periodic fiber at $\check{\varepsilon} = 0.9804$: a) $a = 10^{-2}$, $b = 2.5 \cdot 10^{-3}$; 1) $q = 0.45$, 2) 0.75; b) $a = 2 \cdot 10^{-2}$, $b = 4 \cdot 10^{-4}$, $q = 0.45$.

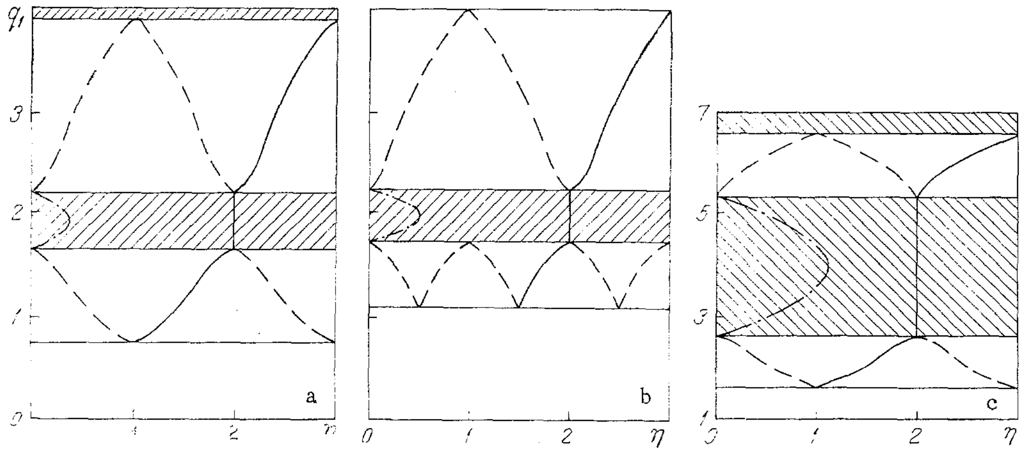

Fig. 3. Brillouin diagram of surface H_{01} wave of gradient periodic fiber at $\check{\varepsilon} = 0.9804$: a) $a = 10^{-2}$, $b = 2.5 \cdot 10^{-3}$, $q = 0.45$; b) $a = 2 \cdot 10^{-2}$, $b = 4 \cdot 10^{-4}$, $q = 0.45$; c) $a = 10^{-2}$, $b = 2.5 \cdot 10^{-3}$, $q = 0.75$.

line (q = const, $\check{\varepsilon}$ = const), which forms an angle φ, equal to $\varphi = \arctan q\varepsilon^{-1}$, with the axis of the conventional longitudinal wave number.

As the gradient parameters of the transverse dielectric permittivity distribution change, the dispersion curve shifts along the straight line of the critical regime of a homogeneous dielectric medium with material characteristics $\mu_0\varepsilon(0)$. Then, as is evident from Fig. 2, the critical point of the waveguide regime moves from the instability zone to the stability zone of the longitudinal Mathieu diagram. Because of this, the waveguide regime within the limits of the first stability zone is completely determined by the critical region of the transverse wave number dispersion characteristic, although previously it was determined by the entire zone. If the amplitude of the longitudinal perturbation of the dielectric permittivity changes, the slope of the lightguide critical regime straight line changes. The effects of change in gradient parameters and dielectric permittivity modulation amplitude can be seen especially clearly in a Brillouin diagram (Fig. 3). With increase in gradient parameters the width of the passband increases, while that of the stopband decreases (Figs. 3a, b). When the modulation amplitude increases (Fig. 3c), the width of the stopband increases, and that of the passband decreases due to the directive properties of the gradient periodic fiber lightguide in the surface wave propagation regime. In the given case this is true not only of Bragg diffraction up to the third order inclusive. Thus, one can pose the problem of optimal synthesis in a manner similar to conventional periodic systems — definitions, following from a specified quality criterion for transverse and longitudinal gradient properties for the required pass and stopbands (regulated distributed inverse feedback). As follows from the results obtained, in open gradient periodic lightguides one can effectively control the propagation regime of surface and quasisurface waves by variation of the transverse spatial profile of the dielectric permittivity.

NOTATION

E_φ, E_r, H_φ, H_r, transverse components of electric and magnetic fields; E_z, H_z, longitudinal electric and magnetic field components; μ_0, ε, magnetic and dielectric permittivities; ω, circular frequency; a, b, spatial gradient parameters; β, spatial frequency; q, dielectric permittivity modulation amplitude; $\varkappa$, σ, internal and external transverse wave numbers; γ, longitudinal wave number; R_1, R_2, special functions; η, surface wave propagation constant; K_1, first order McDonald function; Θ_1, Θ_2, Mathieu functions; $\bar{d}$, dimensionless diameter.

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